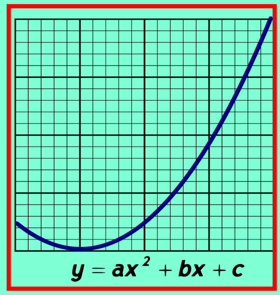
Math 125

Spring 2021

Lecture 27



Class QZ 19

1) Solve $(x-2)^2 = -9$

$u^2 = k$

S.R.M.

$x - 2 = \pm\sqrt{-9}$

$\boxed{x = 2 \pm 3i}$

$\{2 \pm 3i\}$

2) Solve $4x^2 + 9 = 12x$ by the quadratic formula.

$ax^2 + bx + c = 0$

$4x^2 - 12x + 9 = 0$

$a = 4$ $b = -12$ $c = 9$

$b^2 - 4ac = (-12)^2 - 4(4)(9) = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-12) \pm \sqrt{0}}{2(4)}$

$= \dfrac{12 \pm 0}{8} = \dfrac{12}{8} = \boxed{\dfrac{3}{2}}$ $\left\{\dfrac{3}{2}\right\}$

Solve by Completing the Square method:

$3x^2 - 5x - 8 = 0$

Hint: Make leading Coef. 1 by dividing everything by 3.

$\frac{3}{3}x^2 - \frac{5}{3}x - \frac{8}{3} = 0$

$x^2 - \frac{5}{3}x - \frac{8}{3} = 0$

$x^2 - \frac{5}{3}x + \frac{25}{36} = \frac{8}{3} + \frac{25}{36}$

$\frac{1}{2} \cdot \frac{-5}{3} = \frac{-5}{6}, \left(\frac{-5}{6}\right)^2 = \frac{25}{36}$

$\frac{8 \cdot 12}{3 \cdot 12} + \frac{25}{36}$

$= \frac{96}{36} + \frac{25}{36} = \boxed{\frac{121}{36}}$

$\left(x - \frac{5}{6}\right)^2 = \frac{121}{36}$

use S.R.M.

$x - \frac{5}{6} = \pm\sqrt{\frac{121}{36}}$

$x = \frac{5}{6} \pm \frac{11}{6}$

$x = \frac{5}{6} + \frac{11}{6} = \frac{16}{6} = \frac{8}{3}$

$x = \frac{5}{6} - \frac{11}{6} = \frac{-6}{6} = -1$

$\left\{-1, \frac{8}{3}\right\}$

Discuss the type of Solutions for

$(2x - 1)(3x + 4) = -10$

Hint: Use discriminant. $b^2 - 4ac$

write in $ax^2 + bx + c = 0$

$6x^2 + 8x - 3x - 4 + 10 = 0$

$6x^2 + 5x + 6 = 0$

$a = 6 \quad b = 5 \quad c = 6$

$b^2 - 4ac = 5^2 - 4(6)(6) = 25 - 144 = -\#$

$b^2 - 4ac < 0 \Rightarrow$ Two imaginary Solutions

Find a quardratic equation in the form of $ax^2 + bx + c = 0$ with Solutions $\frac{4}{5} \pm \frac{3}{5}i$.

$x = \frac{4}{5} + \frac{3}{5}i$ $\qquad$ $x = \frac{4}{5} - \frac{3}{5}i$

LCD=5 , clear fractions

$5x = 4 + 3i$ $\qquad$ $5x = 4 - 3i$

$5x - 4 - 3i = 0$ $\qquad$ $5x - 4 + 3i = 0$

$$\underbrace{(5x - 4 - 3i)(5x - 4 + 3i)}_{\text{Conjugates}} = 0$$

$$\boxed{(5x-4)^2} - \boxed{(3i)^2} = 0$$

$$25x^2 - 40x + 16 - 9i^2 = 0$$

$$25x^2 - 40x + 16 - 9(-1) = 0$$

$$\boxed{25x^2 - 40x + 25 = 0}$$

Solve by making proper Subs.

$$2x^{\frac{2}{3}} - 5x^{\frac{1}{3}} - 7 = 0$$

Hint: Let $u = x^{1/3}$

$x^{\frac{2}{3}} = \left[x^{\frac{1}{3}}\right]^2$ $\qquad$ $2\left[x^{1/3}\right]^2 - 5\left[x^{1/3}\right] - 7 = 0$

$$2u^2 - 5u - 7 = 0$$

$a = 2 \quad b = -5 \quad c = -7$

$b^2 - 4ac = (-5)^2 - 4(2)(-7)$

$= 25 + 56 = 81$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{81}}{2(2)}$$

$$= \frac{5 \pm 9}{4}$$

$u = \frac{5+9}{4} = \frac{14}{4} = \frac{7}{2}$

$u = \frac{5-9}{4} = \frac{-4}{4} = -1$

$u = \frac{7}{2}$

$x^{1/3} = \frac{7}{2}$

$\sqrt[3]{x} = \frac{7}{2}$

$x = \left(\frac{7}{2}\right)^3$

$\boxed{x = \frac{343}{8}}$

$u = -1$

$x^{1/3} = -1$

$\sqrt[3]{x} = -1$

$x = (-1)^3$

$\boxed{x = -1}$

$$\left\{-1, \frac{343}{8}\right\}$$

$f(x) = \frac{1}{3}(x-3)^2 + 6$

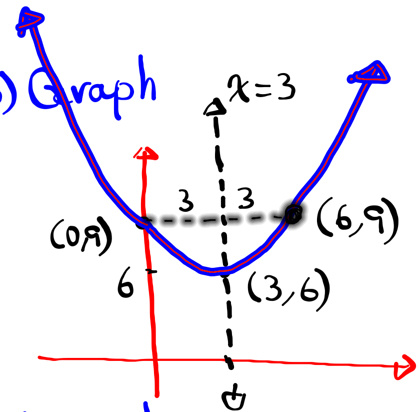
5) Y-Int $(0, 9)$

$f(x) = a(x-h)^2 + k$

1) a, h, k

$a = \frac{1}{3}$, $h = 3$, $k = 6$

6) Graph



2) opens upward $a > 0$

3) vertex $(h, k) = (3, 6)$

4) A.O.S. $x = h$ $x = 3$

7) x-Ints None

8) Domain & Range Domain: $(-\infty, \infty)$ Range: $[6, \infty)$

$f(x) = -x^2 + 4x - 4$

$f(x) = ax^2 + bx + c$

4) A.O.S. $x = h$

$x = 2$

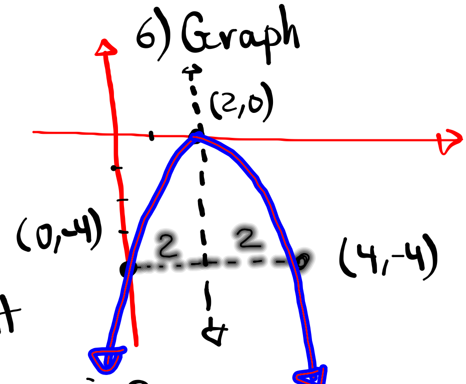
1) a, b, and c

$a = -1$, $b = 4$, $c = -4$

5) Y-Int. $(0, -4)$

2) opens downward, $a < 0$

6) Graph



3) Vertex $(h, k) = (2, 0)$

$h = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

$k = f(h) = -(2)^2 + 4(2) - 4$

$= 0$

7) x-Int

8) Domain & Range $(-\infty, 0]$

Domain $(-\infty, \infty)$

$f(x) = -2(x+3)^2 + 8$

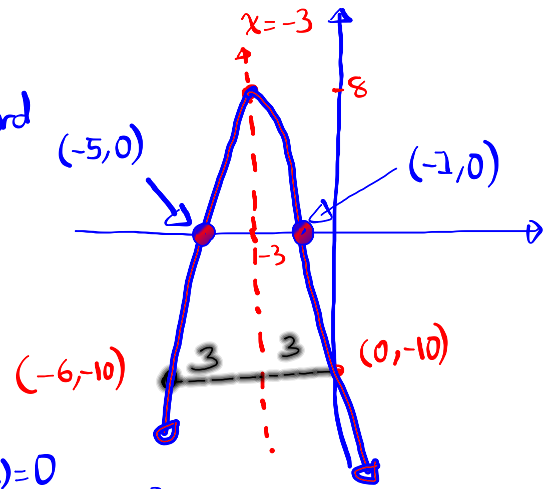
$a = -2$ opens downward

$h = -3$ vertex $(-3, 8)$

$k = 8$

A.O.S. $x = -3$

Y-Int $(0, -10)$



x-Int $y = 0$ $f(x) = 0$

$-2(x+3)^2 + 8 = 0$

$-2(x+3)^2 = -8$

$(x+3)^2 = 4$

S.R.M.

$x + 3 = \pm\sqrt{4}$

$x = -3 \pm 2$

$x = -3 + 2 = -1$

$x = -3 - 2 = -5$

Domain $(-\infty, \infty)$ Range $(-\infty, 8]$

$f(x) = x^2 - 2x - 8$

$a = 1$ $b = -2$ $c = -8$

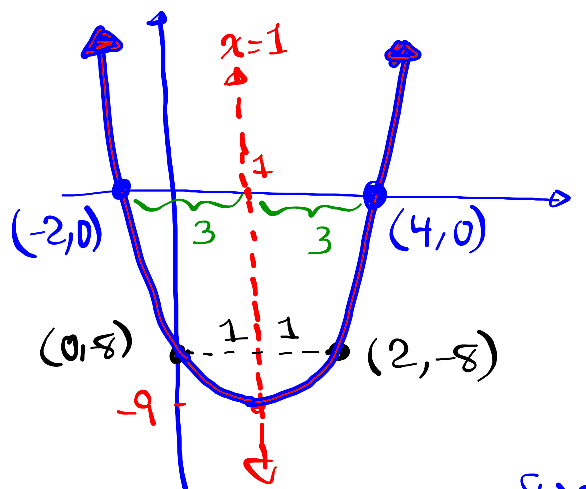
opens upward, $a > 0$

$h = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$

$k = f(h) = 1^2 - 2(1) - 8 = -9$

Vertex $(1, -9)$, A.O.S. $x = 1$

Y-Int $(0, -8)$



x-Int $y = 0$ $f(x) = 0$

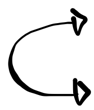
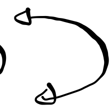
$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ $x = -2$

Sideways Parabola

$x = a(y-k)^2 + h$

$a>0$  , $a<0$ 

Vertex (h, k) A.O.S. $Y = K$ x-Int $(\quad , 0)$

Y-Int $(0, \quad)$ Range: $(-\infty, \infty)$ Domain: See graph

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$x = (y-2)^2 - 4$

$x = a(y-k)^2 + h$

$a = 1$ $h = -4$ $k = 2$

Opens Right, $a>0$

Vertex $(h, k) = (-4, 2)$

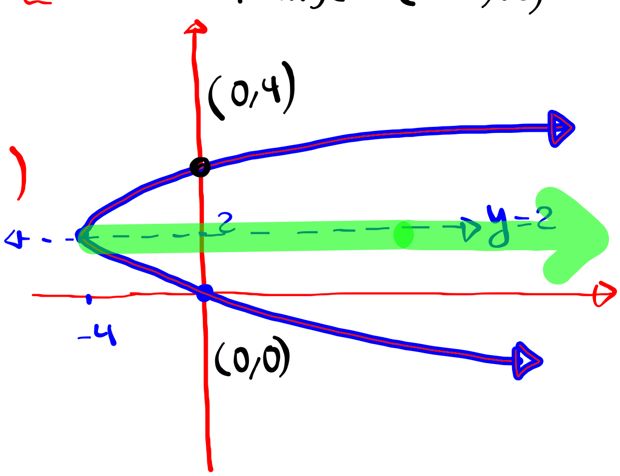
A.O.S. $y = k$ $y = 2$

x-Int $(0, 0)$

Y-Int $(0,0), (0,4)$

Domain $[-4, \infty)$

Range $(-\infty, \infty)$



$x = -\frac{1}{2}(y+2)^2$

5) x-Int $(-2, 0)$

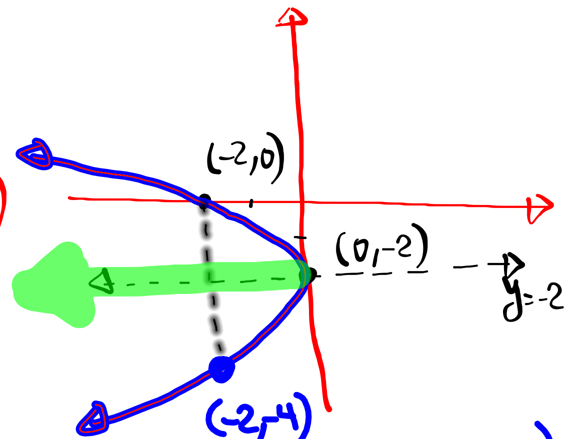
1) $a = \frac{-1}{2}$ $h = 0$ $k = -2$

6) Graph

2) opens left, $a < 0$

3) Vertex $(h, k) = (0, -2)$

4) A.O.S. $y = k$ $y = -2$



Y-Int $(0, -2)$

Domain $(-\infty, 0]$

Range $(-\infty, \infty)$

$x = (y+3)^2 + 2$

Graph

$a = 1$

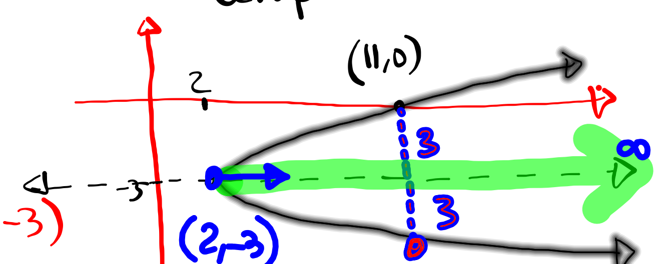
opens Right $a > 0$

$h = 2$

$k = -3$

Vertex $(2, -3)$

A.O.S. $y = -3$



y-Int None

x-Int. $(11, 0)$

Domain: $[2, \infty)$

Range: $(-\infty, \infty)$

$x = -y^2 - 4 \Rightarrow x = -(y-0)^2 - 4$

1) $a = -1$

Direction opens left

$a < 0$

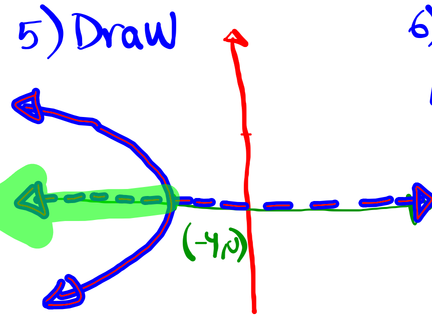
2) h & K $h = -4$ $k = 0$

Vertex $(-4, 0)$

3) A.O.S. $y = K$ $y = 0$

4) x-Int $(-4, 0)$

5) Draw



6) y-Int

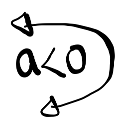
None

7) Domain & Range

Domain: $(-\infty, -4]$ Range: $(-\infty, \infty)$

Sideways Parabola

$$x = ay^2 + by + C \; ; \; a \neq 0$$

Opens  $a > 0$  $a < 0$

Vertex (h, K)

$K = \frac{-b}{2a}$, Plug in K to find h.

A.O.S. $y = K$ x-Int $(\;\; , 0)$ y-Int $(0, \;\;)$

Draw

Range $(-\infty, \infty)$ Domain See graph

$x = y^2 - 6y$

$a = 1 \quad b = -6 \quad c = 0$

Opens Right $a > 0$

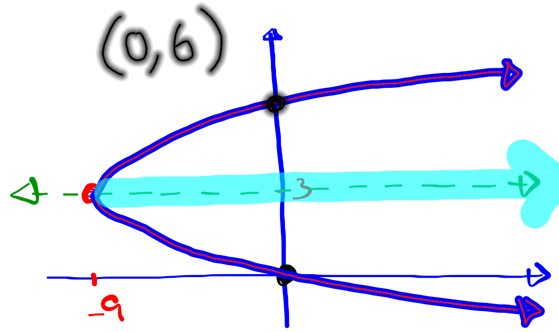
Vertex $(h, k) = (-9, 3)$

$k = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$

$h = 3^2 - 6(3) = 9 - 18 = -9$

A.O.S. $y = k \quad y = 3$

x-Int $(0, 0)$



Domain: $[-9, \infty)$

Range: $(-\infty, \infty)$

$x = -y^2 + 4y$

$a = -1 \quad b = 4 \quad c = 0$

Opens Left, $a < 0$

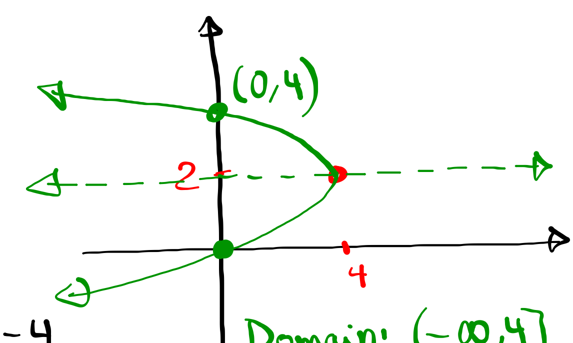
Vertex $(h, k) = (4, 2)$

$k = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

$h = -(2)^2 + 4(2) = -4 + 8 = 4$

A.O.S. $y = k \quad y = 2$

x-Int $(0, 0)$



Domain: $(-\infty, 4]$

Range: $(-\infty, \infty)$

Class QZ 20

1) Solve by Completing the square:

$$x^2 + 14x + 53 = 0$$

2) Discuss the type of Solutions for

$$3x^2 - 4x + 10 = 0$$